Accuracy and Robustness of a 3-D brick Cosserat Point Element (CPE) for finite elasticity

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The theory of a Cosserat point has been used to develop a 3-D brick Cosserat Point Element (CPE) for the numerical solution of problems in finite elasticity. In standard finite element procedures, an approximation of the kinematics (with possible strain enhancements) is assumed to be valid at each point in the element. Then, the stiffness of the element is obtained by integration over the element region. In contrast, the Cosserat approach for an elastic element connects the kinetic quantities to derivatives of a strain energy function. Once the strain energy of the CPE has been specified, the procedure needs no integration over the element region and it ensures that the response of the CPE is hyperelastic. Moreover, since the CPE is a structure, the strain energy function depends on both the reference element geometry and on the three-dimensional strain energy function of the elastic material. Restrictions on the strain energy function of the CPE have been developed which ensure that the CPE reproduces the exact constitutive response to all nonlinear homogeneous deformations. Consequently, the constitutive equations of the CPE analytically satisfy a nonlinear form of the patch test. It is convenient to separate the strain energy function into two parts: one controlling homogeneous deformations and the other controlling inhomogeneous deformations like: bending, torsion and higher-order hourlgassing modes of deformation. Developing a functional form for the strain energy of inhomogeneous deformations has proven to be challenging. Nevertheless, a functional form has been proposed which causes the CPE to be a truly user friendly element that can be used with confidence for problems of finite elasticity including: three-dimensional bodies, thin shells and thin rods. Examples are presented which compare the response of the CPE with that of other elements based on enhanced strain and incompatible mode methods.