Quadrature for Meshless Methods

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Meshless Methods (MM) have been the focus of considerable interest in recent years, especially in the engineering community. This interest was mainly stimulated by difficulties with mesh generation when the Finite Element Method (FEM) is used. It is well-known that creating effective quadrature schemes for MM is an important problem (see, *e.g.,* A stabilized conforming nodal integration for Galerkin mesh free methods, J.-S. Chen, C.-T. Wu, S. Yoon, and Y. You, *Int. J. Numer. Meth. Engng.* 200 1; **50**:435–466); in fact, a problem that needs to be solved if MM are to achieve their potential. In this talk we discuss two developments in the creation of effective quadrature schemes foe MM. We discuss both for the Neumann Problem.

In "Quadrature for Meshless Methods", I. Babuška, U. Banerjee, J. Osborn, and Q. Li, (Int. J, Numer. Methods Appl. Mech. Engng., 2008; 76: 1434-1470) estimates are derived for the energy norm between the exact solution, u, and the quadrature approximate solution, u_h^*, in terms of the parameter h, associated with the family of approximation spaces, and quantities η, τ, and ϵ that measure the relative errors in the stiffness matrix, in the lower order term, and in the right-hand side vector, respectively, due to the quadrature. The major hypothesis is that the quadrature stiffness matrix has zero row sums, which can be achieved by a simple correction of the diagonal elements. Illustrative computations are included.

In "Effect of Numerical Integration on Meshless Methods", Babuška, Banerjee, J. Osborn, and Q. Zhang (preprint), estimates are derived, again for the energy norm between the exact solution, u, and the quadrature approximate solution, u_h^*, in terms of the parameter h, associated with the family of approximation spaces, and quantities $\overline{\eta}$ and $\overline{\tau}$ that measure the basic accuracy of the quadrature schemes. The major hypothesis is that the quadrature scheme satisfies a form of Green's formula. Numerical experiments illuminating the theoretical results are presented. A major improvement in this latter work is that it applies to MM that reproduce polynomials of any order $k \geq 1$, whereas the earlier approach is restricted to $k = 1$.