Combined model and finite element discretization adaptivity for quantities of interest with upper error bounds

Erwin Stein[1], Marcus Rüter[1] and Stephan Ohnimus[2]

[1]Institute of Mechanics and Computational Mechanics (IBNM),
Leibniz Universität Hannover,
Appelstraße 9A, 30167 Hannover, Germany
stein/rueter@ibnm.uni-hannover.de

[2]INPRO Innovationsgesellschaft für fortgeschrittene
Produktionssysteme in der Fahrzeugindustrie mbH,
Hallerstraße 1, 10587 Berlin, Germany
stephan.ohnimus@inpro.de

Abstract

Combined verification and validation is realized in Computational Mechanics by error-controlled discretization and model adaptivity of engineering structures, guaranteeing reliability and achieving computational efficiency. In particular, expansive model adaptivity (with dimensional extension and constitutive upgrades) from coarse to fine mathematical models (in a hierarchical sequence) in related subdomains is reasonable and efficient, in contrast to recursive model adaptivity. Validation needs the incorporation of verified measurements from physical experiments.

We present a deterministic methodology for combined verification and model adaptivity by overall error control of quantities of interest. More precisely, we derive upper error bounds on both the model and the discretization error in terms of the solutions of Neumann problems on the element level. The necessary prolongation of coarse model solutions into the solution space of a fine model for defining a consistent model error is emphasized, which can be achieved on the element level by two different strategies.

The first one is a purely kinematic prolongation of nodal displacements. For the derivation of implicit upper-bound a posteriori error estimates, however, it is necessary that the prolongated solution is orthogonal to the model error, which needs the premultiplication of the prolongation matrices with involutoric orthogonal matrices.

The second strategy uses a kinematic prolongation of the external loads which are then used to solve an additional variational problem thus yielding a prolongated solution which a priori fulfills the required orthogonality relation.

Furthermore, due to the Galerkin orthogonality, also the finite element solution of the coarse model is orthogonal to the discretization error and it is shown that no prolongation of this finite element solution is required[1].

Finally, illustrative numerical examples are presented where the error estimator is applied to linear elasticity with a Reissner-Mindlin plate as the coarse model and the fully 3D theory as the fine model.

References

1. Stein E, Rüter M, Ohnimus S. Error-controlled adaptive goal-oriented modeling and finite element approximations in elasticity, *Computer Methods in Applied Mechanics and Engineering*, **196**, 3598–3613, 2007.